Extended NJL model with eight quark interactions

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We review the effect on vacuum properties and magnetic catalysis, of adding the most general 0-spin 8-quark ($8q$) interactions to the combined $SU(3)$ flavor Nambu–Jona-Lasinio (NJL) and 6-quark ($6q$) 't Hooft interaction Lagrangian.

Since the 8-quark interaction vertices have been introduced[1)] in the combined 3-flavor NJL and 't Hooft determinantal Lagrangians[2)−5)] to resolve the instability of the vacuum associated with the $6q$ terms,[6)] a series of phenomenoligical implications for the hadron phenomenology has been studied. From the two types of $8q$ interactions considered below, the first ($\sim g_1$) violates the OZI rule, and induces the most relevant effects.

$$
\begin{aligned}
\mathcal{L}^{(1)}_{8q} &= 8\, g_1 \left[(\bar{q}_i P_R q_m)(\bar{q}_m P_L q_i)\right]^2 = \frac{g_1}{32}\left[\mathrm{tr}(S-iP)(S+iP)\right]^2 = \frac{g_1}{8}\left(S_a^2+P_a^2\right)^2 , \\
\mathcal{L}^{(2)}_{8q} &= \frac{g_2}{16}\mathrm{tr}\left[(S-iP)(S+iP)(S-iP)(S+iP)\right]. \qquad (0{\cdot}1)
\end{aligned}
$$

The trace is taken over the flavor indices $i,j=1,2,3$ of $S_{ij}=S_a(\lambda_a)_{ij}=2\bar{q}_j q_i$, $P_{ij}=P_a(\lambda_a)_{ij}=2\bar{q}_j(i\gamma_5)q_i$, and λ_a denote the Gell-Mann matrices for $a=1,...,8$ and the flavor singlet λ_0. After bosonization of the theory in stationary phase approximation one obtains[1)] a set of 3-coupled equations for the quark condensates $\langle \bar{q}_i q_i \rangle = h_i/2$

$$
G h_i + M_i - m_i + \frac{\kappa}{16}\sum_{j\neq k\neq i} h_j h_k + \frac{g_1}{4}\, h_i \sum_{j=u,d,s} h_j^2 + \frac{g_2}{2}\, h_i^3 = 0, \qquad (0{\cdot}2)
$$

where M_i, m_i denote the constituent and current quark masses respectively, G is the $4q$ and κ the $6q$ coupling strengths. These equations must be solved self-consistently with the gap equations $h_i(M_i)+\frac{N_c M_i}{2\pi^2}J_0(M_i^2)=0$ with J_0 denoting the quark one loop tadpole. Being of cubic order equations (0·2) can be chosen to have a single real root within a constrained set of values for the coupling constants: $g_1>0$, $g_1+3g_2>0$, $G>\frac{1}{g_1}\left(\frac{\kappa}{16}\right)^2$. The resulting effective potential is bounded from below. The stability constraints allow for a domain of sufficiently small values of the g_1, g_2 couplings, together with an overall good fit of the low lying meson nonet characteristics.[7)] Furthermore, noting that the combination $\xi = G + g_1(h_u^2+h_d^2+h_s^2)/4$ multiplying h_i in (0·2) also appears in all but the singlet-octet mixing scalar meson mass expressions[8),7)] one infers that an increase in the strength of the $8q$ terms can be tuned to a decrease in the $4q$ strength G, leaving the spectra almost unaffected. In fact noticeable changes occur only in the σ-meson mass, which decreases with increasing g_1.[7)] One sees also that g_1 will steer the value of the effective coupling ξ due to any change in the condensates caused by external parameters such as the temperature, the chemical potential, the electromagnetic field, etc. It is exactly in

these cases that one expects the $8q$ interactions to unfold all their potential and most importantly, without affecting the vacuum results. As an illustration we consider the quark fields minimally coupled to a constant magnetic field H. In $(2+1)$ and $(3+1)$ D a constant magnetic field H catalyzes the dynamical symmetry breaking of the NJL model with $4q$ intercations generating a fermion mass even as $G \to 0$ and the symmetry is not restored at any arbitrarily large H.[9] We obtain that the inclusion of higher order multiquark interactions, in particular the $8q$ ones, leads to new effects.[10] In the Landau gauge and in the chiral limit the gap equation in presence of H reads

$$-\frac{2\pi^2 h(M)}{\Lambda^2 N_c} = f(M^2; \Lambda, |QH|), \tag{0·3}$$

where h(M) is determined through (0·2). The couplings G, κ, g_1, g_2 are contained only in the l.h.s. of (0·3), and H is present only in $f(M^2; \Lambda, |QH|)$ (see[10] for explicit form of f). Q stands for mean quark charge and Λ is the cutoff. The function f is singular as $M \to 0$ for $H \neq 0$. For $\kappa = g_1 = g_2 = 0$ the l.h.s. reduces to the constant $\frac{2\pi^2}{\Lambda^2 N_c G}$, after dividing by M, thus eliminating the trivial solution at $M = 0$. The gap equation has always a nontrivial solution even in the subcritical regime (G not strong enough to break chiral symmetry when $H = 0$). The higher multiquark interactions however distort the l.h.s. which cause the order parameter to increase sharply (a secondary magnetic catalysis) with increasing strength of the field at the characteristic scale $H \sim 10^{19} G$. A new phase of massive quarks emerges and becomes the stable configuration at that scale.[10] Recently the implications of $8q$ interactions in a strong magnetic background have been considered in an extended $SU(2)$ flavor PNJL model to study confinement and chiral symmetry restoration, and in the dressed Polyakov loop analysis of the phase diagram of hot quark matter.[11]

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References

1) A. A. Osipov, B. Hiller, J. da Providencia, Phys. Lett. B **634** (2006), 48.
2) Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961);
3) G. 't Hooft, Phys. Rev. D **14**, 3432 (1976); G. 't Hooft, Phys. Rev. D **18**, 2199 (1978).
4) V. Bernard, R. L. Jaffe and U.-G. Meissner, Phys. Lett. B **198**, 92 (1987);
5) H. Reinhardt and R. Alkofer, Phys. Lett. B **207**, 482 (1988).
6) A. A. Osipov, B. Hiller, V. Bernard , A. H. Blin, Annals of Phys. **321** (2006), 2504.
7) A. A. Osipov, B. Hiller, A. H. Blin, J. da Providencia, Annals of Phys. **322** (2007), 2021.
8) B. Hiller, J. Moreira, A.A. Osipov, A.H. Blin, Phys. Rev. D **81** (2010), 116005.
9) P. Klevansky and R. H. Lemmer, Phys. Rev. **39**, 3478 (1989). K. G. Klimenko, Theor. Math. Phys. **89**, 211 (1991). I. V. Krive and S. A. Naftulin, Sov. J. Nucl. Phys. **54**, 897 (1991). V. P. Gusynin, V. A. Miransky, I. A. Shovkovy, PRL **73**, 3499 (1994).
10) A. A. Osipov, B. Hiller, A.H. Blin, J. da Providencia, Phys. Lett. B **650** (2007), 262; B. Hiller, A. A. Osipov, A.H. Blin, J. da Providencia, SIGMA **4** (2008), 024.
11) R. Gatto and M. Ruggieri, Phys. Rev. D **83** (2011), 034016, R. Gatto and M. Ruggieri, Phys. Rev. D **82** (2010), 054027.